MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023 (TERM ID 2220)

TMA1401 – INTRODUCTION TO DISCRETE MATHEMATICS AND LINEAR ALGEBRA

(All sections / Groups)

06 JULY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTION TO STUDENT

1. This question paper consists of 4 printed pages (inclusive of the front page) with 4 questions only. Page 4 is the appendix for the logical equivalence laws.
2. Attempt **ALL FOUR** questions. The distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.
4. Show **ALL** of your working steps clearly.

QUESTION 1 **[TOTAL: 10 MARKS]**

a) Let the universal set $U = \{x \in Z^+ \mid x \leq 11\}$, $A = \{2,3,6\}$ and $B = \{x \mid x \text{ is a prime number}\}$.

i) List all the elements of B [1 mark]

ii) Find $A \Delta B$ [2 marks]

iii) Find $P(A)$ [1 mark]

b) Let R be relation defined on the set $A = \{2,4,8\}$ such that

$$R = \{(a,b) \in A \times A \mid (a+b) \text{ divisible by } 3\}$$

i) List all elements of $A \times A$ [1 mark]

ii) List all elements of R [1 mark]

iii) Is R symmetric? Justify your answer. [1 mark]

iv) Is R antisymmetric? Justify your answer. [1 mark]

v) Is R transitive? Justify your answer. [1 mark]

vi) Is R a function? Justify your answer. [1 mark]

QUESTION 2 **[TOTAL: 10 MARKS]**

a) Consider the points $P(3, 1, 2)$, $Q(1, 3, 1)$ and $R(2, 4, 5)$.

i) Find the vector $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. [2 marks]

ii) Find a vector perpendicular to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ [2 marks]

b) Consider the following matrix A:

$$A = \begin{bmatrix} 6 & 16 \\ -1 & -4 \end{bmatrix}$$

i) Find the eigenvalues for the matrix A. [2 marks]

ii) For each eigenvalue obtained in (i), find its eigenvector. [4 marks]

QUESTION 3 **[TOTAL: 10 MARKS]**

a) Use truth table to determine whether the statement is tautology, contradiction, or contingency.

$$p \wedge q \rightarrow \neg p \vee q$$ [2.5 marks]

Continued ...

b) Use equivalence laws to show that $(q \to \neg p) \wedge (p \to \neg r)$ and $p \to \neg(q \vee r)$ are logically equivalent. [2.5 marks]

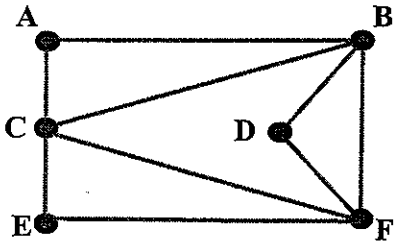
c) Use mathematical induction to show that

$$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3} \text{ for all integers } n \geq 1.$$

[5 marks]

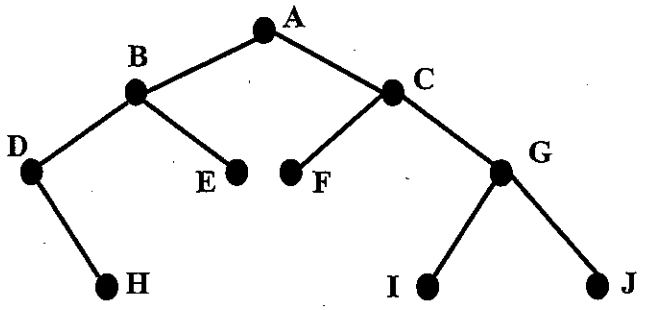
QUESTION 4 **[TOTAL: 10 MARKS]**

a) Given the following graph:



Use the **breadth first search** algorithm to find a spanning tree for the above graph. Take vertex **D** as the root and apply alphabetical order if there is more than one choice in the path. Clearly show each step of how the algorithm is performed and present your answer in a table with three columns, the first column is the number of steps, the second column is the queue for the search algorithm and the third column is the edges of the resulting spanning tree. [5 marks]

b) Find the visitation order for the tree below using **in-order** traversal.



[2 marks]

Continued ...

c) Twenty tickets, numbered 1, 2, 3, …,20 is sold to 20 shop customers for lucky draw. Three different prizes (1st, 2nd, and 3rd prize) are awarded. How many ways are there to award the prizes if

(i) there are no restrictions? [1 mark]

(ii) the people holding tickets 6 and 9 both win prizes and people holding 2 and 12 do not win prizes? [2 marks]

Appendix

List of Logical Equivalence Laws

Conversion of Implication: $p \rightarrow q \Leftrightarrow \neg p \vee q$

Conversion of Equivalence: $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

Double Negation: $\neg\neg p \Leftrightarrow p$

DeMorgan	:	(i)	$\neg (p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$		
		(ii)	$\neg (p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$		
Domination	:	(i)	$p \wedge F \Leftrightarrow F$	(ii)	$p \vee T \Leftrightarrow T$
Negation	:	(i)	$p \wedge \neg p \Leftrightarrow F$	(ii)	$p \vee \neg p \Leftrightarrow T$
Identity	:	(i)	$p \wedge T \Leftrightarrow p$	(ii)	$p \vee F \Leftrightarrow p$
Commutative	:	(i)	$p \wedge q \Leftrightarrow q \wedge p$	(ii)	$p \vee q \Leftrightarrow q \vee p$
Idempotent	:	(i)	$p \vee p \Leftrightarrow p$	(ii)	$p \wedge p \Leftrightarrow p$
Distributive	:	(i)	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$		
		(ii)	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$		
Associative	:	(i)	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r \Leftrightarrow p \vee q \vee r$		
		(ii)	$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r \Leftrightarrow p \wedge q \wedge r$		
Absorption	:	(i)	$p \vee (p \wedge q) \Leftrightarrow p$		
		(ii)	$p \wedge (p \vee q) \Leftrightarrow p$		

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